

**UNIFIED COUNCIL**

Foundation for success

**UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD****CLASS - 9****Question Paper Code : 40119****KEY**

1	2	3	4	5	6	7	8	9	10
D	A	C	A	A	D	A	B	C	D
11	12	13	14	15	16	17	18	19	20
A	C	A	B	C	B	B	C	A	A
21	22	23	24	25	26	27	28	29	30
D	D	B	C	B	D	C	A	A	D
31	32	33	34	35	36	37	38	39	40
A,B,C,D	A,B,C	A,B,C,D	A,B,C,D	A,C,D	A	A	C	B	A
41	42	43	44	45	46	47	48	49	50
C	D	D	D	C	B	A	D	D	A

EXPLANATIONS**MATHEMATICS**

01. (D) $m - \sqrt{1} + m + \sqrt{1} = 4 + 20$

$$2m = 24 \Rightarrow m = \frac{24}{2} = 12$$

02. (A) Let $\sqrt[4]{124 + 32\sqrt{15}} = \sqrt{\sqrt{a} + \sqrt{b}}$

Square twice to get

$$a + b = 124 \text{ and } ab = 3840$$

$$a = 64, b = 60$$

$$\text{Simplify } \sqrt{8 + 2\sqrt{15}} = \sqrt{5} + \sqrt{3}$$

03. (C) Given $p(x) = 2x^3 - 11x^2 + 17x - 6$
 $p(-1) = 2(-1)^3 - 11(-1)^2 + 17(-1) - 6 = -2 - 11 - 17 - 6$

$p(-1) \neq 0 \Rightarrow (x + 1)$ is not a factor of $p(x)$

$$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$$

$$= 16 - 44 + 34 - 6$$

$p(2) = 0 \Rightarrow (x - 2)$ is a factor of $p(x)$

04. (A) $x = 40^\circ + 65^\circ = 105^\circ$

05. (A) Given $3x + 4x + 5x = 144$ m
 $12x = 144$ m
 $x = 12$ m
 $\therefore a = 36$ m, $b = 48$ m, $c = 60$ m and
 $s = 72$ cm

Area of the triangle =

$$\begin{aligned} & \sqrt{72 \times (72-36)(72-48)(72-60)} \\ &= \sqrt{72 \times 36 \times 24 \times 12} \text{ m}^2 \\ &= \sqrt{2 \times 36 \times 36 \times 24 \times 12} \\ &= 36 \times 24 \text{ m}^2 \\ &= 864 \text{ m}^2 \end{aligned}$$

06. (D) Area of $\Delta AOB = \frac{1}{4}$ area of the 11 gm
ABCD
 $= \frac{1}{4} \times 52 \text{ cm}^2$
 $= 13 \text{ cm}^2$

07. (A) Volume $= \frac{4}{3} \pi r^3 = 38808$
 $\frac{4}{3} \times \frac{22}{7} \times r^3 = 38,808 \text{ cm}^3$
 $r^3 = 38808 \times \frac{3 \times 7}{4 \times 22}$
 $r = \sqrt[3]{21^2 \times 21}$
 $r = 21 \text{ cm}$

Surface area of a sphere

$$\begin{aligned} &= 4\pi r^2 = 4 \times \frac{22}{7} \times 21 \times 21 \\ &= 5544 \text{ cm}^2 \end{aligned}$$

08. (B) $\angle CBA = 180^\circ - 100^\circ = 80^\circ$
But $\angle CBA + \angle ADC = 180^\circ$
 $80^\circ + \angle ADC = 180^\circ$
 $\angle ADC = 100^\circ$
But $\angle CDF + \angle ADC = 180^\circ$
 $\angle CDF + 100^\circ = 180^\circ$
 $\angle CDF = 80^\circ$

09. (C) $p(1) = (1)^3 - (1)^2 + 1 + 1 = 1 - 1 + 1 + 1 = 2$
 $p(-1) = (-1)^3 - (-1)^2 + (-1) + 1 = -1 - 1 - 1 + 1 = 2$

$$\frac{p(1) + p(-1)}{2} = \frac{2 + (-2)}{2} = 0$$

10. (D) $5y + y + 2y + y = 180^\circ [\because AB \parallel CD]$
 $9y = 180^\circ$

$$y = \frac{180^\circ}{9} \Rightarrow y = 20^\circ$$

11. (A) From the second equation

$$\begin{aligned} x &= y - 1 \\ \text{Substitute into the first equation} \\ 3(y-1) - 5y &= -1 \\ 3y - 3 - 5y &= -1 \\ -2y &= 2 \\ y &= -1 \\ \text{Then, } x &= -1 - 1 = -2 \end{aligned}$$

12. (C) A linear equation in two variables always has infinite solutions (line).

13. (A) Given $AB = AC \Rightarrow ACB = ABC = x$
In BOC , given $OB = OC$ $OCB = OBC = y$
But $ABO + OBC = x$
 $ABO + y = xy$
 $ABO = x - y$
similarly we can prove $ACO = x - y$

$$\therefore ABO : ACO = (x - y) : (x - y) = 1 : 1$$

14. (B) The cube's space diagonal $(a\sqrt{3})$ equals the sphere's diameter $(3\sqrt{3} \text{ cm})$

Solve for side length $a = 3 \text{ cm}$.

$$\text{Volume} = a^3 = 27 \text{ cm}^3$$

15. (C) ABCD is a cyclic quadrilateral.
 $\therefore \angle A + \angle C = 180^\circ$
 $\angle A + 100^\circ = 180^\circ$
 $\angle A = 80^\circ$

$$\text{In } \triangle ABD, 80^\circ + 30^\circ + \angle ADB = 180^\circ$$

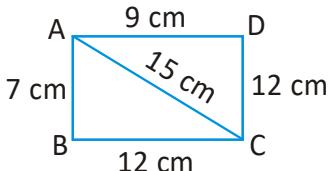
$$\angle ADB = 180^\circ - 110^\circ = 70^\circ$$

16. (B) Volume of the pyramid

$$= \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times 8 \times 8 \times 12 \text{ cm}^3 = 256 \text{ cm}^3$$

17. (B) Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$



$$\text{where } s = \frac{a+b+c}{2} = \frac{7\text{cm}+12\text{cm}+15\text{cm}}{2} = \frac{34\text{cm}}{2} = 17 \text{ cm}$$

$$\text{Area } (\triangle ABC) = \sqrt{17 \times 10 \times 5 \times 2} \text{ cm}^2 = 10\sqrt{17} \text{ cm}^2$$

$$\text{In } \triangle ACD, 15^2 = 12^2 + 9^2 \Rightarrow \angle D = 90^\circ$$

$$\therefore \text{Area of } \triangle ACD = \frac{1}{2} \times AD \times DC$$

$$= \frac{1}{2} \times 9 \times 12^6 \text{ cm}^2 = 54 \text{ cm}^2$$

\therefore Area of the quadrilateral ABCD

$$= (10\sqrt{17} + 54) \text{ cm}^2$$

18. (C) Given $p(x) = \frac{2x+1}{x-2}$

$$\therefore p\left(\frac{2x+1}{x-2}\right) = \frac{2\left(\frac{2x+1}{x-2}\right) + 1}{\left(\frac{2x+1}{x-2}\right) - 2}$$

$$= \frac{\left[\frac{4x+2+(x-2)}{x-2}\right]}{\left[\frac{2x+1-2(x-2)}{x-2}\right]}$$

$$= \frac{(4x+2+x-2)}{(x-2)} \times \frac{(x-2)}{(2x+1-2x+4)} = \frac{5x}{5} = x$$

19. (A) If $x = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

$$\Rightarrow x - 2 = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$$

Cubing on both sides, we get

$$(x-2)^3 = \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)^3$$

$$\Rightarrow x^3 - 6x(x-2) - 8 = 2 + 6(x-2) + 4$$

$$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 6(x-2)$$

$$\Rightarrow x^3 - 6x^2 + 6x = -6 + 8$$

$$\Rightarrow x^3 - 6x^2 + 6x = 2$$

20. (A) Given AOB is an equilateral triangle

$$\therefore \angle OAB = \angle OBA = \angle AOB = 60^\circ$$

$$\therefore \angle OAD = \angle OBC = 90^\circ - 60^\circ = 30^\circ$$

In $\triangle AOD$ and $\triangle BOC$

$$\overline{OA} = \overline{OB} \quad (\because \text{side \& sides of equilateral triangle})$$

$$\angle OAD = \angle OBC \quad [\because \text{Angle}]$$

$$\overline{AD} = \overline{BC} \quad (\because \text{side \& side of a square})$$

$$\therefore \triangle AOD \cong \triangle BOC \quad [\because \text{SAS congruency}]$$

$$\therefore OD = OC \quad [\because \text{CPCT}]$$

$$\text{In } \triangle COD, OD = OC \Rightarrow \angle OCD = \angle ODC = 42^\circ$$

$$\therefore \angle ODA = 90^\circ - \angle ODC = 90^\circ - 42^\circ = 48^\circ$$

$$\text{In } \triangle AOD, \angle ODA + \angle OAD + \angle DOA = 180^\circ$$

$$48^\circ + 30^\circ + \angle DOA = 180^\circ$$

$$78^\circ + \angle DOA = 180^\circ$$

$$\angle DOA = 180^\circ - 78^\circ = 102^\circ$$

21. (D) $(x^4 - x^3 + x - 1) = x^3(x-1) + 1(x-1)$

$$= (x-1)(x^3 + 1)$$

$$= (x-1)(x+1)(x^2 - x + 1)$$

$$x^4 + x^2 + 1 = x^4 + x^2 + 1 + x^2 - x^2$$

$$= (x^4 + 2x^2 + 1) - x^2$$

$$= (x^2 + 1)^2 - x^2$$

$$= (x^2 + x + 1)(x^2 - x + 1)$$

$$\therefore \text{HCF} = (x^2 - x + 1)$$

22. (D) Given $a = 11 \text{ cm}$

$b = 60 \text{ cm}, c = 61 \text{ cm}$

$$\therefore s = \frac{a+b+c}{2}$$

$$= \frac{(11+60+61) \text{ cm}}{2} = \frac{132 \text{ cm}}{2} = 66 \text{ cm}$$

Area of $\triangle ABC$

$$= \sqrt{66 \times (66-11)(66-60)(66-61)} \text{ cm}^2$$

$$= \sqrt{66 \times 55 \times 6 \times 5} \text{ cm}^2$$

$$= 11 \times 6 \times 5 \text{ cm}^2 = 330 \text{ cm}^2$$

$$\frac{1}{2} \times \text{biggest side} \times \text{shortest altitude}$$

$$= 330 \text{ cm}^2$$

$$\frac{1}{2} \times 61 \text{ cm} \times \text{shortest altitude} = 330 \text{ cm}^2$$

$$\therefore \text{Shortest altitude} = 330 \text{ cm}^2 \times \frac{2}{61} \text{ cm}$$

$$= 10.819 \text{ cm}^2 = 10.82 \text{ cm}^2$$

23. (B) $\angle PTQ = 90^\circ$ [\because Angle in a semicircle]

TQRS is a cyclic quadrilateral

$$\therefore \angle QTS + \angle SRQ = 180^\circ$$

$$\therefore x + y = \angle PTQ + \angle QTS + \angle SRQ = 90^\circ + 180^\circ = 270^\circ$$

24. (C) $(0,0), (-1,1), (1,1), (0,2)$ are four ordered pairs satisfies the given condition that $x^{2020} + y^2 = 2y$

25. (B) Let the height be ' x '

$$\therefore \text{Radius} = \frac{2}{3}x = \frac{5}{3}x$$

$$\text{Given } 2\pi rh = 4620 \text{ cm}^2$$

$$\Rightarrow 2 \times \frac{22}{7} \times \frac{5x}{3} \times x = 4620 \text{ cm}^2$$

$$x^2 = \frac{4620}{210} \times \frac{10521}{221} \text{ cm}^2 \times \frac{1}{2} \times \frac{7}{221} \times \frac{3}{5}$$

$$x^2 = (21\text{cm})^2$$

$$\therefore x = 21 \text{ cm}$$

$$\therefore \text{Radius} = \frac{5}{3}x = \frac{5 \times 21}{3} = 35 \text{ cm}$$

$$\text{Total surface area} = 2\pi r (h + r)$$

$$= 2 \times \frac{22}{7} \times 35 \times (21 + 35) \text{ cm}$$

$$= 220 \times 56 \text{ cm}$$

$$= 12320 \text{ cm}^2$$

26. (D) Area of the sector = CSA of a cone

$$\frac{1}{2} \pi (2\sqrt{3})^2 = \pi r \times (2\sqrt{3})$$

$$\frac{1}{2} \frac{(2\sqrt{3})^2}{(2\sqrt{3})} = r$$

$$r = \sqrt{3}$$

$$h = \sqrt{l^2 - r^2} = \sqrt{(2\sqrt{3})^2 - \sqrt{3}^2}$$

$$= \sqrt{12 - 3} = \sqrt{9} = 3$$

$$\text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \times (\sqrt{3})^2 \times 3 = 3\pi \text{ cm}^3$$

27. (C) Maximum length of pencil = diagonal of a cuboid

$$= \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{(12)^2 + (9)^2 + 8^2} \text{ cm}$$

$$= \sqrt{144 + 81 + 64} \text{ cm}$$

$$= \sqrt{289} \text{ cm}$$

$$= 17 \text{ cm}$$

$$28. (A) \text{ LHS} = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$$

29. (A)
$$\left(\sqrt[3]{x} + \sqrt[3]{y}\right)\left(\sqrt[3]{x^2} - \sqrt[3]{xy} + \sqrt[3]{y^2}\right)$$

$$= \left(\sqrt[3]{x}\right)^3 + \left(\sqrt[3]{y}\right)^3$$

$$= (x + y)$$

30. (D) Given angle = 45° \Rightarrow Height of empty part = Diameter = 6 cm
 \therefore Volume of the remaining water

$$= \pi \times 3 \times 3 \times 11 \text{ cm}^3 - \frac{1}{2} \times \pi \times 3 \times 3 \times 6^3 \text{ cm}^3$$

$$= 99\pi \text{ cm}^3 - 27\pi \text{ cm}^3$$

$$= 72\pi \text{ cm}^3$$

$$= 72\pi \text{ ml} \quad [\because 1 \text{ cm}^3 = 1 \text{ ml}]$$

MATHEMATICS - 2

31. (A,B,C,D)

Bring everything to one side:

$$a^2 + b^2 + c^2 - ab - bc - ca = 0$$

This is equivalent to:

$$\frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$

This implies:

$$(a-b)^2 = 0, (b-c)^2 = 0, (c-a)^2 = 0$$

$$\Rightarrow a = b = c$$

32. (A,B,C)

In a rhombus, kite and square the diagonal are always perpendicular.

33. (A,B,C,D)

$$4(0) + 5(2.8) = 14$$

$$\therefore (0, 2.8) \text{ lies on } 4x + 5y = 14$$

$$4(3) + 5(0.4) = 12 + 2 = 14$$

$$\therefore (3, 0.4) \text{ lies in } 4x + 5y = 14$$

$$4(-1.5) + 5(4) = -6 + 20 = 14$$

$$\therefore (-1.5, 4) \text{ lies on } 4x + 5y = 14$$

$$4(5) + 5(-1.2) = 20 - 6 = 14$$

$$\therefore (5, -1.2) \text{ lies on } 4x + 5y = 14$$

34. (A,B,C,D)

All options are true.

35. (A,C,D)

$$\angle A = \angle B + 15^\circ$$

$$\angle C = \angle B - 30^\circ$$

$$\text{But } \angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + 15^\circ + \angle B + \angle B - 30^\circ = 180^\circ$$

$$3\angle B - 15^\circ = 180^\circ$$

$$3\angle B = 195^\circ$$

$$\angle B = \frac{195^\circ}{3} = 65^\circ$$

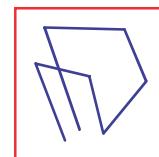
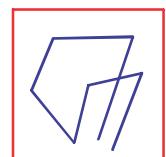
$$\angle A = 65^\circ + 15^\circ = 80^\circ$$

$$\angle C = 65^\circ - 30^\circ = 35^\circ$$

$$\angle A + \angle B = 80^\circ + 65^\circ = 145^\circ$$

REASONING

36. (A) R is the sister of P, which means R and P are siblings. P is the son of Q, so Q is the parent of P, and therefore also the parent of R. T is given as the mother of Q. Since Q is R's parent, T is R's grandparent. Specifically, as T is the mother of R's parent, R is T's granddaughter.



37. (A)

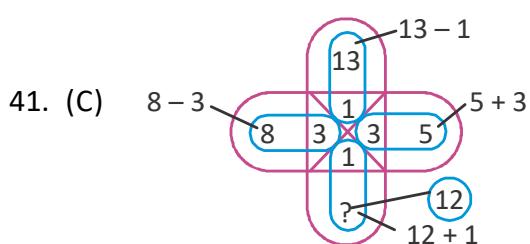
38. (C) One of the dots lies in the region common to the circle the triangle and the rectangle only, another dot lies in the region common to the circle and the triangle only and the third dot lies in the region common to the circle, the square and the triangle only. In each of the figures (1), (2) and (4) there is no region common to the circle, the triangle and the rectangle only. Only fig. (3) consists of all the three types of regions.

39. (B) Initially facing East

90° clockwise \rightarrow South

45° anti-clockwise \rightarrow halfway between East and South = South - East

40. (A) If in the word a letter is o th letter from the beginning of the English alphabet then in the code, the corresponding letter is the $(n + 1)$ th letter from the end of the alphabet.



42. (D) From both the figures we find that numbers 1, 2 and 4 dots appear adjacent to 6. Thus, the number 5 dots will appear opposite to 6. Therefore when 6 is at the bottom, then 5 will be at the top.

43. (D)
- (A) $8 - 43 \div 6 \times 1 = 4$ (X)
- (B) $7 \times 6 \div 2 + 3 - 6 = 5$ (X)
- (C) $10 + 7 - 3 \div 2 = 4$ (X)
- (D) $2 \times 5 - 6 + 2 = 6$ (✓)

44. (D) The arrow is being rotated clockwise around the circle, in the sequence A-C-E, B-D-F etc. Therefore, figure D is the odd one out because the arrow has been reflected instead of rotated.

45. (C) We are counting triangles in an equilateral triangular grid with 4 rows. The standard formula is:

$$\text{Total triangles} = \left[\frac{n(n+2)(2n+1)}{8} \right]$$

Here, $n = 4$ (number of rows). Substituting

$$\begin{aligned} \text{Total triangles} &= \frac{4(4+2)(2 \times 4 + 1)}{8} \\ &= \frac{4 \times 6 \times 9}{8} = \frac{216}{8} = 27 \end{aligned}$$

CRITICAL THINKING

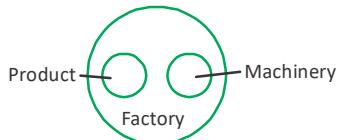
46. (B) The university cannot provide grants (Statement II), so the college decides to become autonomous. The students oppose this decision (Statement I) because they don't want the change.

Cause → Statement II

Effect → Statement I

47. (A) $C \rightarrow S$ (child) $\rightarrow U$ (grandchild). Since S is male, C must be S 's parent. No info giving C 's gender explicitly. But P is father-in-law of F implies F 's spouse is child of P ; that spouse is S ? Wait F 's children include N and U , whose father must be F 's spouse. S is father of U and $N \rightarrow S$ is F 's husband. Therefore S is husband of F . Then S is son of C , so C is parent of S and thus grandparent of U . Gender of C : since S is son of C , C could be mother or father. But options: Grandmother, Grandfather, Father, Mother. C is grandparent of U ; male or female? Given typical naming, since P is father-in-law of F (P male) and P is father of F 's spouse; spouse is S ; so P is father of S . But we also have S is son of C , so C must be spouse of P (other parent of S). So C is mother of S (since P male father). Thus C is grandmother of U .

48. (D)



49. (D) Statement I alone gives number of rams but not how many are black. Statement II alone gives total black sheep ($40\% \text{ of } 50 = 20$) but not how many of those are female (ewes). Together still don't tell how many of the black sheep are ewes.

50. (A) The new math department chairperson is chosen based on seniority. From the information given, Ms. Wilson has more seniority than Ms. Brown but less than Mr. Tony, while Mr. Rohan is more senior than Ms. Wilson but less than Mr. Tony. Arranging them by seniority, we get: Ms. Brown < Ms. Wilson < Mr. Rohan < Mr. Tony. Since Mr. Tony does not want the position, the most senior eligible person is Mr. Rohan. Therefore, Mr. Rohan will be appointed as the new chairperson.